Free Vibration Analysis of Orthotropic (SSSS) Non-Homogeneous Parallelogram Plate with Varying Bi-Dimensional Thickness and Thermal Effect

A Synopsis

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Manoj Kumar Dhiman
(Registration Number: H15IECPHD03)

Under the Supervision of

Dr. Ashish Kumar Sharma
HOD
Department.of Mathematics
IEC University, Baddi (HP), India

Department of Mathematics,
IEC University, Baddi (HP), INDIA

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Introduction

Plate

A plate is a plane structural component having very small thickness as compared to other surface dimensions. Sometimes thickness vary, but most of times it is constant. The dynamic forces applied to the plates are perpendicular to the surface of plate. The impact of these forces on plate leads to bending and twisting moments. On the basis of thickness to width ratio plates are classified into two types: thin plates and thick plates. In case of thin plates, the thickness to breadth ratio of plate is less than 0.1, otherwise the plate is considered to be thick plate. In recent years, applications and importance of plate structure in engineering and industry have remarkably increased. Due to lighter structures and cost-effective advantages, plates are widely used in foundation slabs, bridge slabs and modern technology such as ship building, automobile industry, aircraft building, turbines etc. The analysis of plates is mainly concern with the parameters like:

- Plate's geometry (rectangular, parallelogram, square, circular, polygon etc.).
- Behavior of material (isotropic, orthotropic) and its thickness.
- Load applied (static, dynamic, mechanical, hydrostatic, thermal etc.).
- Boundary conditions (clamped, simply supported, free).

Vibrations of plate

The study in the field of vibration plays an important role in the branch of applied science and engineering. By vibration we mean a movement of the particle of an elastic or rigid body which repeats itself periodically. Vibration is the main cause of collapse in plates with sprung mass. In the modern era, we can not neglect the effect of vibrations as all engineering machines and structures experience vibrations. Since vibrations directly effects the life and work power of the machine, therefore knowledge about the first few modes of vibration is essential and necessary to a mechanical engineer, before finalizing a design.

In vibration problems the thermal dependence of frequency on plates of different shapes are of great importance. In previous years, a lot of research has been done in the field of

vibration of plates. Vibration of plates having different geometry and behavior such as orthotropic/isotropic, homogeneous/non-homogeneous and either considering or not considering the effect of temperature and thickness variation have been studied by number of authors. The analysis of temperature dependent vibration of plate is very significant in the design of power plant turbines, nuclear reactors and other structure works at elevated temperature. Thermally induced vibrations of non-uniform plates are also very useful for research workers in aeronautical, chemical and nuclear engineering. Since modern engineering requires very accurate solutions, therefore the subject matter of the present study has been restrained to a study of vibration problems of non-homogeneous and orthotropic parallelogram plates of uniform and/or inconsistent thickness, having simple and/or mixed boundary conditions taking into account the result of shear deformation under different temperature field. In industries the materials exposed to high temperature generally deviate from Hooke's law and behaves visco-elastically. The elastic and viscous behavior of material depends mainly on frequency and temperature. Consequently, the vibration analysis has become very important from the point of view of designing a structure to be familiar in advance about its reaction. So that the essential measure to manage the structural vibrations and its amplitudes can be taken.

Visco-elastic Plate/ Material

Viscoelasticity is the response of material that shows the characteristics of viscous fluid and an elastic solid both under deformation. A visco-elastic material is a material that posses the viscoelasticity to regain its original shape after the application of stress, but does it at slow rate in order to oppose the vibrations in next cycle. The temperature and frequency are two major parameters on which the behavior of a material depends. The properties of visco elastic materials like rubber, concrete, soil, wood,cement etc. may show linear visco-elasticity but metals and their alloys at high temperature show non linear visco-elasticity according to their mechanical behavior. Since vibrations affect the efficiency and strength of machine as well as durability of the system, it is necessary for the engineers to develop more and more accurate design of the structure to give better performance and efficiency. Therefore investigation of the vibration problems in visco-elastic theory is required in many structural applications of these materials due to thermal effect.

The property of these types of materials lies between elastic and plastic. Theory of elasticity and plasticity explains the techniques of deformation of most engineering materials. Both the theories as applied to metals and alloys are based on tentative studies of the relation between stress and strain. Under the effect of external forces, theory of elasticity defines the stress, strain and displacement in an elastic body. Upto certain level All structural materials posses the property of elasticity. It means, if external forces producing deformation of a structure do not exceed to a certain level, the deformation disappears with the removal of the external forces. For various engineering disciplines the purpose of studying elasticity is to analyses the stresses and displacements of structural and machine elements in the range and thereby to check the sufficiency of their strength, stiffness and stability. The theory of plasticity is basically the mathematical study of stress and strain in plastically distorted solids. This follows the well-established precedent set up by the 'theory of elasticity'. The relation of plastic and elastic properties of metals to crystal structures and cohesive forces belongs to the subject now known as 'metal physics'.

Visco- elastic material i.e. Duralium is the composition of

a) Aluminum: 95%,
b) Copper: 4%,
c) Magnesium: 0.5%,
d) Manganese: 0.5%.

Duralium alloys are comparatively soft, ductile and extruded or drawn into array of shapes and products. Because of its high strength per unit weight and comparatively light in weight than steel, suit it for aircraft manufacture.

Plate with variable Thickness

Plates of various thicknesses are frequently applied in recent technologies such as: Buildings, Aeronautical industry, Automobile industry, Power stations, Marine Engineering. These are specially applied in manufacture of rockets, missiles and wings, tails and fins of high-speed airplanes. With the advancement of technology, the necessitate to find the result of different vibration of plates having variable thickness by visco-elastic medium have intensified. In view of the fact that the plates with tapered

geometry are normally applied to form the structures, therefore plates having inconsistent thickness are of vast use in an extensive array of manufacturing applications.

Plate with Thermal effect

The significance of thermal effect on the mechanics of solid structures has been increased in recent years due to the quick growth in the field of space engineering, aeronautical engineering, civil structures, ocean structures, nuclear field and machine designs etc. Due to variation in temperature, non homogeneity occurs in material, which can not be neglected. Therefore it is necessary to study the temperature variations on plates. Since the area of present research is very vast, therefore we have made some assumptions as follows:

- It is assumed that all the four edges of the plate are clamped.
- It is assumed that temperature varies linearly in x and y directions.
- It is also assumed that deflection is linearly small.
- Viscoelasticity is assumed to be of 'Kelvin' type.
- It is assumed that the Poisson ratio is constant throughout.
- Young Modulus becomes the function of space variable due to thermal effect.

Present work is contribution towards the analysis of thermal vibrations of orthotropic visco-elastic plates of variable and uniform thickness. The frequency equations and displacement functions for non-homogeneous plates of variable and uniform thickness and their solutions for various boundary conditions under the influence of different types of varying transient temperature fields has been derived. With the help of modern techniques, frequency of vibration for different modes is evaluated for different values of parameters (such as taper constant, thermal gradient). The mathematical calculations will be processed by considering the substance constants of alloy 'Duralium'.

Literature Review

The review of allied literature as well as previous research work is of vital importance in research endeavour. Under this review of related literature an attempt has been made to draw some meaningful guidelines for the present research work. Numerous researchers have studied the vibration behavior of plates (homogeneous or non-homogeneous) with variable thickness and either taking / not taking into account the effect of temperature. After review of various research papers and literature, it is evident that not much work has been done in the field of vibration problems with variable thickness of plates in two directions. Therefore present work intends to carry out studies in the area of two-dimensional variation in temperature and thickness.

Leissa [1] in his monograph and a series of review articles [4] studied vibration of plates.

Dokainnish and Kumar [2] analyzed Vibration of orthotropic parallelogramic plates with variable thicknesses.

Tomar and Gupta [3] studied the constant temperature effect on free axis symmetric vibrations of orthotropic rectangular plate having linearly varying thickness. First and second modes of vibration frequencies of a plate have been computed using different combinations of boundary conditions for various values of temperature gradient and taper constant.

Gupta, Kumar and Gupta [5] analyzed the effect of vibration on orthotropic parallelogram plate having varying thickness linear in one-dim. Plate is supposed to be clamped and the principal equation in differential form has been solved with the help of variable separation technique. Rayleigh-Ritz technique has been applied with two term deflection function to derive frequency equation. First and second modes of vibration frequencies of a plate having parallelogram geometry has been calculated for various values of thermal gradient, aspect ratio, skew angle etc.

Gupta and Khanna [6] analyzed the effect of free vibration on clamped rectangular plate with exponentially varying thickness in both directions. Rayleigh-Ritz technique has been applied with two term deflection function to derive frequency equation. First and second modes of vibration frequencies of a plate having parallelogram geometry has

been calculated for various values of thermal gradient, aspect ratio, skew angle etc.

Gupta, Kumar and Gupta [7] studied the problem of vibration of visco-elastic parallelogram plate having parabolically varying thickness. Plate is supposed to be clamped on all the four edges and variation in thickness is parabolic in one direction. The principal equation in differential form has been solved with the help of variable separation technique. First and second modes of vibration frequencies of a clamped plate having parallelogram geometry has been calculated for various values of thermal gradient, aspect ratio, skew angle and taper constant.

Gupta and Kumar [8] presented the problem of effect of temperature on vibrations of plate having parallelogram geometry with varying thickness and temperature linear along x-axis. The principal equation in differential form has been solved with the help of variable separation technique. Rayleigh-Ritz technique has been applied with two term deflection function to derive frequency equation. First and second modes of vibration frequencies of a clamped plate having parallelogram geometry has been calculated for various values of thermal gradient, aspect ratio, skew angle and taper constant.

Gupta and Kumar [9] have discussed the effect of temperature on vibration of parallelogram plate in which thickness and temperature is varying linearly in one direction. Rayleigh-Ritz method has been applied to obtain the frequency equations. First and second modes of vibration frequencies of a plate having parallelogram geometry has been calculated for various values of thermal gradient, aspect ratio, skew angle and taper constant.

Gupta, Kumar, Khanna and Kumar [10] studied the thermal effect on vibration of parallelogram plate having linearly varying thickness in both directions. Rayleigh-Ritz technique has been applied with two term deflection function to derive frequency equation. First and second modes of vibration frequencies of a plate having parallelogram geometry has been calculated for various values of thermal gradient, aspect ratio, skew angle etc.

Gupta, Kumar & Kaur [11] studied vibration effect on visco-elastic orthotropic parallelogram plate having linearly varying thickness in two directions. The principal equation in differential form has been solved with the help of variable separation technique. Rayleigh-Ritz technique has been applied with two term deflection function to

derive frequency equation. First and second modes of vibration frequencies of a plate having parallelogram geometry has been calculated for various values of thermal gradient, aspect ratio, skew angle etc.

Gupta, Rana and Gupta [12] reviewed the effect of temperature on vibration of non-homogeneous parallelogram plate having linearly varying thickness in two directions. It has been assumed that density varies linearly for a plate of non-homogenous material. The principal equation in differential form has been solved with the help of variable separation technique. An equation was derived by Rayleigh-Ritz method with two terms deflection function. First and second modes of vibration frequencies of a plate having parallelogram geometry has been calculated for various values of thermal gradient, aspect ratio, skew angle etc.

Kumud and Gupta [13] studied the temperature effect on vibration of non-homogeneous parallelogram plate with thickness varying linearly. Rayleigh-Ritz technique has been applied with two term deflection function to derive frequency equation. First and second modes of vibration frequencies of a plate having parallelogram geometry has been calculated for various values of thermal gradient, aspect ratio, skew angle etc.

Gupta and Kumud [14] studied temperature effect on vibration of parallelogram plate having parabolic varying thickness. Density and temperature is supposed to vary linearly. The principal equation in differential form has been solved with the help of variable separation technique. Rayleigh-Ritz technique has been applied with two term deflection function to derive frequency equation. First and second modes of vibration frequencies of a plate having parallelogram geometry has been calculated for various values of thermal gradient, aspect ratio, skew angle etc.

Khanna & Arora [15] studied the effect of vibrations on non-homogeneous parallelogram plate with sinusoidal thickness variation and bi-linear temperature variation. Rayleigh-Ritz technique has been applied with two term deflection function to derive frequency equation.

Gupta & Kumar [16] analyzed the effect of vibration on visco-elastic plate having parallelogram geometry with linear thickness variation along x-axis and parabolic thickness variation along y-axis. The principal equation in differential form has been

solved with the help of variable separation technique for clamped plate. Rayleigh-Ritz technique has been applied with two term deflection function to derive frequency equation. First and second modes of vibration frequencies of a plate having parallelogram geometry has been calculated for various values of thermal gradient, aspect ratio, skew angle etc.

Pratibha Arora [17] studied vibration of non-homogeneous parallelogram plate having bi-linear variation of temperature and thickness. While due to non-homogeneity density vary parabolically in one direction. Rayleigh-Ritz technique has been applied with two term deflection function to derive frequency equation of clamped plate. First and second modes of vibration frequencies of a plate having parallelogram geometry has been calculated for various values of thermal gradient, aspect ratio, skew angle etc.

Sharma, Raghav & Kumar [18] studied vibration on non-homogeneous parallelogram plate with thermal gradient. Temperature and thickness of plate is supposed to vary bi-linear. Due to non-homogeneity density vary linear in one direction. For clamped plate, Rayleigh-Ritz technique has been applied with two term deflection function to derive frequency equation. First and second modes of vibration frequencies of a plate having parallelogram geometry has been calculated for various values of thermal gradient, aspect ratio, skew angle etc.

Sharma & Sharma [19] discussed the vibration effect by mathematical modeling on non-homogenous parallelogram plate. Temperature and thickness of plate is supposed to vary bi-linear. Due to non-homogeneity density varies linearly in one direction. For clamped plate, Rayleigh-Ritz technique has been applied with two term deflection function to derive frequency equation. First and second modes of vibration frequencies of a plate having parallelogram geometry has been calculated for various values of thermal gradient, aspect ratio, skew angle, taper constant.

Justification

The justification of any research problem depends upon its application in a variety of fields. Keeping in view, the researcher has chosen the topic entitled **"Free Vibration Analysis of Orthotropic (SSSS) Non-Homogeneous Parallelogram Plate with Varying Bi-Dimensional Thickness and Thermal Effect"**

From the literature review it is observed that adequate work has been done in the field of temperature variation in one direction as compared to variation of temperature in two directions. Currently a lot of research has been done to justify the applications of vibration in the field of engineering such as design of power plant machinery, machines, foundations, structures etc. Due to disproportionate vibrations, the instruments fixed with machinery may lose its accuracy. On the whole, design considerations related to modifying the vibrational behavior of automatic devices and structures are important in the present scenario. Besides that, we know that the plates of inconsistent thickness are broadly applied in recent technologies such as:

- Mechanical/Civil Engineering
- Aeronautical industry
- Automobile Industry
- Nuclear Power Stations
- Marine Engineering

In the aeronautical industry, plates having thermal constant and inconsistent thickness are of enormous significance because of their usefulness in airplane wings. Since engineers have become more aware regarding the importance of an elastic behavior of material. Therefore, we cannot neglect the non- homogeneity occurred in material due to temperature variation on plates. Mathematical formulations are done so that the problem becomes more useful and important for practical use in engineering field. One of the most interesting result in this work concerns with engineers and designers who'll be benefitted with this study in the planning, modification and control techniques of designing and they can learn the observations, analysis, for mechanical, civil engineering structures and aeronautical systems that are optimized against the effects of vibration.

Objectives

The main objectives of the present research are:

- To analyze bi-linearly varying temperature effect on vibration of non-homogeneous parallelogram orthotropic plate with inconsistent thickness linearly in two dimensions.
- To analyze bi-parabolically varying temperature effect on vibration of non-homogeneous parallelogram orthotropic plate with inconsistent thickness linearly in two dimensions.
- To analyze bi-linearly varying temperature effect on vibration of non-homogeneous parallelogram orthotropic plate with inconsistent thickness parabolically in two dimensions.
- To analyze bi-parabolically varying temperature effect on vibration of non-homogeneous parallelogram orthotropic plate with inconsistent thickness parabolically in two dimensions.
- To analyze exponentially varying temperature effect on vibration of non-homogeneous parallelogram orthotropic plate with inconsistent thickness exponentially in two dimensions.

Proposed Methodology

From time to time, various researchers tried to find the exact or series solutions to vibrations of plates problems. The problem becomes more complex due to the field engaged by the plate and the particular edge conditions. When the elastic constant (young's modulus, poison's ratio, density, modulus of rigidity) of a material varies from point to point in the material, it is said to be elastically non-homogeneous.

In a Kelvin model; the following assumption are made

- The deflection is considered from the middle plane of the plate.
- The upward force which is acting on the plate is directly proportional to the deflection.

Therefore approximate methods are needed for such type of plate problems. We'll apply the Rayleigh-Ritz method to resolve the frequency equations of the plates of variable thickness. Rayleigh-Ritz methods are efficient methods to handle the vibration of plate. problems. Rayleigh-Ritz method is expanded form of Rayleigh's Method. In Rayleigh-Ritz method, it is considered that by satisfying some boundary conditions, value closer to the exact value can be obtained with the help of a linear combination of several functions. This method concludes that the maximum kinetic energy of a system is equal to the maximum strain energy under the condition of no damping and by using a single function approximation the estimated value of the lowest frequency can be obtained. All the calculations or computational work and results to be presented in graphs using MATLAB.

Thesis Outline

In the proposed thesis an attempt will be made to study the vibration under the effect of temperature and non-uniform varying thickness on non-homogeneous parallelogram plate. The tentative chapter wise plan of the thesis has been proposed as under:

Chapter 1:

Introduction and Preliminaries.

Chapter 2:

To analyze bi-linearly varying temperature effect on vibration of non-homogeneous parallelogram orthotropic plate with inconsistent thickness linearly in two dimensions.

Chapter 3:

To analyze bi-parabolically varying temperature effect on vibration of non-homogeneous parallelogram orthotropic plate with inconsistent thickness linearly in two dimensions.

Chapter 4:

To analyze bi-linearly varying temperature effect on vibration of non-homogeneous parallelogram orthotropic plate with inconsistent thickness parabolically in two dimensions.

Chapter 5:

To analyze bi-parabolically varying temperature effect on vibration of non-homogeneous parallelogram orthotropic plate with inconsistent thickness parabolically in two dimensions.

Chapter 6:

To analyze exponentially varying temperature effect on vibration of non-homogeneous parallelogram orthotropic plate with inconsistent thickness exponentially in two dimensions.

Chapter 7:

Conclusions and scope for further research.

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RESEARCH SCHOLAR

Manoj Kumar Dhiman
Research Scholar,
Registration Number: H15IECPHD03
Deptt. of Mathematics,
IEC University, Baddi (HP), India

SUPERVISOR

Dr. Ashish Kumar Sharma
HOD,
Deptt. of Mathematics,
IEC University, Baddi (HP), India